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PAVLOVSKY'S THEORY FOR PHREATIC LINE AND SLOPE STABILITY

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the simplicity and ease of application of the theory, it is a valuable aid in soils work. So that its use may become more widespread, description of the theory and its use is illustrated in the examples that follow.

Professor Pavlovsky in his theoretical development considered only embankments with impervious foundations, because from the hydraulic point of view an embankment on an impervious foundation gives the highest elevation of the phreatic line. However, the theory is also approximately applicable to dams on pervious foundations. In the latter case, the seepage water appears in the foundation as well as in the fill itself. The rate of percolation and the volume of flow through the compacted fill will differ from the rate and volume through the foundation largely because of the difference in type and properties of the material. A dam designed for an impervious base, but to be constructed on a porous one, will have a line of saturation lower than assumed, and the seepage pressure on the downstream slope will be less. This will add stability and safety to the embankment section or will permit economic modifications in the design. The derivation of the formulas for the seepage flow through the homogenous dam and for locating the theorectical line of saturation was simplified by considering a section of an earth fill of unit thickness, divided into three characteristic parts which are shown in Figure 1 and are called upstream, central, and downstream parts, respectively. The general hydraulic properties of the percolation water through each of these parts are analyzed separately.

In the upstream part of the earth dam, after water penetrates the upstream slope, the water starts to flow into this part of earth fill (as shown in Figure 1, between the upstream slope and Section W-W). During percolation of water through this part, the water meets resistance which requires some energy to overcome. Therefore, in the second part of its path, in the central part, it has much less energy. As in the motion of ground water, the kinetic energy is negligible compared with the potential energy. The potential head referred to datum 0-0 is represented by H_w, the depth of the water on the upstream side. Therefore, the loss of head $\underline{a}$ (in the upstream part) is expressed by the difference between H_w and $\underline{h}$, where $\underline{h}$ is the remainder of the total potential energy of the water after percolating through the upstream part of the earth dam.

From Section W-W (Figure 1), the percolating water does not have a uniform motion through the soil, and the line of saturation in the central part of the dam will be a descending curve and will intersect the surface of downstream slope at point M_o.

From hydraulic considerations, the position of this point will be located above the tail water surface. The position of M_o is utilized as a starting point in analyzing the stability of the downstream slope of an earth fill as will be seen in the second part of this article.

In studying the general hydraulic properties of the downstream part of dam, the following statements apply: The potential energy of water for the entire dam is $H_w - h_o$. If the outlet point, M_o, of the line of saturation is at the elevation of the tail water (Figure 1), the potential energy for the downstream part is zero, and there would be no flow in the downstream part. But since there is flow in the downstream part, it must be concluded that the outlet point M_o must be above the level of the tail water. Hence a_o is introduced to represent the height of the outlet point M_o above the elevation of the tail water, and the following equation can be written

$$y_o = a_o + h_o$$

From the above statement, the following conclusion can be drawn: The downstream part of the earth dam has two zones of percolation flow: that is, the upper zone, with a discharge of percolation flow q_1, which goes to the atmosphere, and the second, below the first zone under level of the tail water with a discharge q_2. Therefore, the total discharge through the section of the earth dam of unit thickness is

$$q = q_1 + q_2$$

In previous discussion, the case was considered in which the depth of tail water $h_o > 0$, but in practice h_o is usually zero; that is, there is usually no water at the downstream side of the earth dam. In this case, the mathematical treatment concerning the upstream part, and the central part of the earth dam will be the same as in previous case. But in the downstream part, the position of the outlet point M_o of the line of saturation will change and will be at an elevation a_o above the base line (Figure 2). This elevation is necessary to overcome the hydraulic resistance during the percolation of water through the downstream part of the dam.

Concerning the general hydraulic properties of the percolation through an earth dam, it is necessary to consider the direction of the flow lines at the entrance to the body of the dam at the upstream part. The upstream slope represents the equipotential line. It is known that the equipotential lines are normal to the flow lines. So it is evident that the flow lines in an earth dam must be normal to the surface of the upstream slope at the time of entrance to the upstream part. During the percolation flow in the upstream part, the flow lines will change their direction and at the time of entrance to the boundary of the central part, they will generally choose a nearly horizontal direction. This conclusion applies to the line of saturation itself, which represents the upper flow line of the percolating water. From description of the main scheme of percolation of water through a homogenous earth dam on an impervious foundation, the flow in the body of the dam can be considered to be a hydraulic system with three divisions; i.e., the upstream part plus the central part plus the downstream part. These three parts represent one hydraulic unit, and through each part the same amount of water, q, percolates. Derivations of the final usable equations pertaining to these problems are presented in the appendix of this paper. However, the study and more detail concerning the other types of earth fill section may be found in "Technical Memorandum No. 383," Bureau of Reclamation.

Applying the Dupuit formula and Darcy's law for the flow of water through a homogenous earth dam, the general equation for the line of saturation for the central part of the dam was derived, the mathematical expression of which is:

$$y^2 = h^2 - \frac{2qx}{k} \qquad (1)$$

where,

y = vertical ordinate
x = positive distance from W-W to point considered
k = coefficient of permeability
q = percolation discharge per unit length of the dam and
h = the elevation of point M_1, above the base line

For the percolation discharge through the body of an earth dam, three equations were developed by Pavlovsky; i.e., for the upstream part, the central part, and the downstream part. When $h_o > 0$ as in Figure 1 these equations have the following expressions:

$$\frac{q}{k} = \frac{(H_d - h - d_o)}{m} \log_e \frac{Hd}{Hd - h} \qquad (2)$$

$$\frac{q}{k} = \frac{1}{2S} \left[h^2 - (a_o + h_o)^2 \right] \qquad (3)$$

$$\frac{q}{k} = \frac{a_o}{m_1} \left[1 + \log_e \frac{a_o + h_o}{a_o} \right] \qquad (4)$$

These three equations include four unknowns:

q, a_o, h, and S.

However, one of the four unknowns, S, can be easily determined from the hydraulic boundary of the earth dam (Figure 1) and

$$S = b + m_1 \left[H_d - (a_o + h_o) \right] \qquad (5)$$

Equations (2) through (5) determine the ordinates of the main points of the line of saturation, M, M_1, and M_o (Figure 1), and together with Equation (1) the intermediate points between M_1 and M_o.

From study of equations (2 to 4) the position of the line of saturation in an earth dam does not depend upon the coefficient of permeability of the earth material from which the dam is constructed, but depends only on the hydraulic boundaries of the structure.

After simultaneous solution of the above equations, determining the value k, and knowing the coefficient of permeability, k, the percolation discharge per unit length of dam can be computed.

The first part of the line of saturation in the upstream part of the dam, between the point M and M_1, cannot be plotted by points, but the above method gives the position of the two points, M and M_1, where the first point is determined by the intersection of the upstream surface water with the slope of the dam and the second by the value of h at M_1.

The line of saturation between the points M_1 and M_o can be plotted completely using Equation 1, and the curve M to M_1 must give a smooth connection to have a common tangent at point M_1 with the curve from M_1 to M_o.

The tangent at point M_1 of the line of saturation forms an acute angle (θ) with the axis of the abscissa and is described by

$$\tan \theta = \frac{q}{kh} \qquad (6)$$

This explanation is sufficient for practical plotting of the line of saturation in the body of the homogenous dam because a more accurate plotting of the line of saturation is not necessary in designing the section of an embankment.

For simplification and ease of application of the percolation flow equations, the following designation is introduced in Equation 4.

$$A = 1 + \log_e \frac{a_o + h_o}{a_o} \qquad (7)$$

Then Equation 4 will be

$$\frac{q}{k} = \frac{a_o}{m_1} A \qquad (8)$$

Substituting the above expression in Equation 3 and solving for $\underline{h}$ we have

$$h = \sqrt{\frac{2Sa_o}{m_1} A + (a_o + h_o)^2} \qquad (9)$$

and substituting the expression for $\underline{S}$ from Equation 5, Equation 9 becomes

$$h = \sqrt{2a_o A \left[\frac{b}{m_1} + H_d - (a_o + h_o)\right] + (a_o + h_o)^2} \qquad (10)$$

Equation 2 can be simplified by having

$$\log_e \frac{H_d}{H_d - h} = E \qquad (11)$$

then Equation 2 becomes

$$\frac{q}{k} = \frac{H_d - h - d_o}{m} E \qquad (12)$$

Substituting Equation 8 in Equation 12 we obtain the following expression

$$\frac{m}{m_1} a_o A = (H_d - h - d_o) E \qquad (13)$$

The careful examination of Equations 7, 10, and 11 indicates that the values of $\underline{A}$, $\underline{h}$, and $\underline{E}$, (at given H_d, b, h_o, d_o, m, m_1) are functions of a_o; in other words

$$A = f_1(a_o)$$
$$h = f_2(a_o) \qquad (14)$$
$$E = f_3(a_o)$$

Equation 13 has only one unknown, a_o, the other unknowns, $\underline{h}$, $\underline{S}$, and $\frac{q}{k}$ have been resolved by the previous mathematical operations. Equation 13 can now be solved by the cut and try method or by a graphical method. The graphical method is the much simpler and is obtained by plotting in Figure 3:

$$F_1(a_o) = \frac{m}{m_1} a_o A \qquad (15)$$

$$F_2(a_o) = (H_d - h - d_o) E \qquad (16)$$

After plotting the data of both functions between resonable limits, the desired value of a_o will be determined as the abscissa of the intersection of the curves of the two functions.

As soon as the value a_o is found, the other unknowns can be determined as follows: The value $\underline{S}$ can be computed from Equation 5. After substituting a_o in Equation 7, the value for A can be computed, and by using Equation 9,

the value of h can be found. Finally by using Equation 8, the percolation flow per unit length of dam can be determined, provided the coefficient of permeability is known. All above discussion applies when $h_o > 0$. When $h_o = 0$, the equations can be used in a similar manner as follows:

Equations 2 and 12 remain unchanged, and basic Equations 3, 4, and 5 reduce to the following:

$$\frac{q}{k} = \frac{h^2 - a^2}{2S} \qquad (3.1)$$

$$\frac{q}{k} = \frac{a_o}{m_1} \qquad (4.1)$$

$$S = b + m_1 \left[H_d - a_o\right] \qquad (5.1)$$

Also Equation 10 can be reduced and is

$$h_1 = 2a_o \left(\frac{b}{m_1} + H_d - a_o\right) + a_o^2 \qquad (10.1)$$

where h_1 indicates the special case.

Substituting in Equation 12 the value of $\frac{q}{k}$ from Equation 4.1 we have

$$\frac{a_o m}{m_1} = (H_d - h_1 - d_o)\ E \qquad (13.1)$$

which can be solved by cut and try or graphical means as for the case in which $h_o > 0$. After determining the value of a_o, S can be found by Equation 5.1, the value h_1 by Equation 10.1, and from Equation 4.1 percolation flow per unit length of dam can be determined from

$$q = \frac{a_o k}{m_1} \qquad (8.1)$$

The following numerical example illustrates the computations where h_o is greater than zero, showing in Table 1 an arrangement of the mathematical expressions which simplify the computation. A similar arrangement is easily set up where h_o is equal to zero.

Numerical Example

Given: Height of the dam H_d = 100.0 feet

Freeboard d_o = 10.0 feet

Depth of water in reservoir H_w = 90.0 feet

Width of the crest b = 30.0 feet
Upstream slope Ctn = m = 3.00
Downstream slope $Ctn_1 = m_1$ = 2.75

Depth of tail water = h_o = 45.0 feet

Coefficient of permeability k = 2.0 feet per year
(from laboratory tests)

Required to find:

(1) The vertical distance from the surface of tail water to the outlet point (M_o) of the line of saturation . . . . . a_o

(2) The horizontal distance from Section W-W (see Figure 1) to the outlet point (M_o) . . . . . S

(3) The maximum ordinate at the intersection of the line of saturation with Section W-W . . . . . h

(4) The percolation discharge through the unit length of the dam. q

Note: The numerical values are taken for this example only and should not be considered as recommended values for any design.

First, we must determine the value a_o. For this purpose, use Equation 13 and substitute different values of a_o, until the left function, $F_1(a_o)$, is approximately equal to the right function, $F_2(a_o)$, as is shown underlined in the right hand column of Table 1. This gives the required value for a_o.

The value of a_o may be found also by graphical method by plotting values of $F_1(a_o)$ and $F_2(a_o)$ from Table 1 against the value of a_o in Figure 3 and observing the abscissa of the intersection of the two curves.

Table 1

Item No	Equations	Simplified equations	Computation columns			
1	a_o	a_o	6.0	10.0	6.95	
2	$y_o = h_o + a_o$	45.0 + (1)	51.0	55.0	51.95	
3	$\frac{y_o}{a_o}$	$\frac{(2)}{(1)}$	8.50	5.50	7.475	
4	$A = 1 + \log_e \frac{y_o}{a_o}$	$A = 1 + \log_e$ (3)	3.14007	2.70475	3.01156	
5	$F_1(a_o) = \frac{m}{m_1} a_o A$	$F_1(a_o) = 1.09(1)(4)$	20.54	29.48	<u>22.81</u>	
6	$(\frac{b}{m_1} + H_d) - y_o$	110.91 - (2)	59.91	55.91	58.96	
7	$2a_o A \left[(\frac{b}{m_1} + H_d) - y_o\right]$	2(1).(4).(6)	2257.46	3024.45	2468.11	
8	y_o^2	$(2)^2$	2601.00	3025.00	2698.0	
9	$h^2 = 2a_o A \left[(\frac{b}{m_1} + H_d) - y_o\right] + y_o^2$	h^2 = (7) + (8)	4858.46	6049.45	5166.91	
10	$h = \sqrt{2a_o A \left[(\frac{b}{m_1} + H_d) - y_o\right] + y_o^2}$	$h = \sqrt{(9)}$	69.70	77.78	71.88	
11	$H_d - h$	100.0 - (10)	30.30	22.22	28.12	
12	$\frac{H_d}{H_d - h}$	$\frac{100.0}{(11)}$	3.30	4.50	3.56	
13	$E = \log_e \frac{H_d}{H_d - h}$	$E = \log_e$ (12)	1.19392	1.50403	1.26976	
14	$H_d - h - do$	(11) - 10.0	20.30	12.22	18.12	
15	$F_2(a_o) = \left[(H_d - h) - do\right] E$	$F_2(a_o)$ = (14).(13)	24.24	18.38	<u>23.01</u>	OK

Note: a_o = 6.95 feet = 7.0 feet (see also Figure 3).

After determining a_o, we can proceed to determine the other unknowns. By substituting in Equation 5, we have

$$S = 30.0 + 2.75 \quad [100.0 - (7.0 + 45.0)] \quad = 162.0 \text{ feet}$$

and from Equation 9 we get

$$h = \sqrt{\frac{2 \times 162 \times 7.0 \times 3.01}{2.75} + (7.0 + 45.0)^2} = 72.02 \text{ feet}$$

by Equation 8

$$\frac{q}{k} = \frac{7.0}{2.75} \times 3.01 = 7.66 \text{ and}$$

$q = 7.66 \times 2.0 = 15.3$ cubic feet per year per linear foot of dam or 0.31 gallon per day per linear foot of dam

From the calculated values of $\underline{h}$ and $\frac{q}{k}$, by using Equation 1 the trace of the line of saturation can be plotted for the central part of the dam between the points M_1 and M_o.

For this numerical example, the equation of the curve is

$$y^2 = (72.02)^2 - 2\,(7.66)\,x$$

$$= 5186.88 - 15.32\,x$$

The plotted data are shown in Figure 4.

Author's note: In the designing of an earth dam, two types of movement of moisture in soil can be considered. These are: (a) The movement of water which is dependent on pore space (in saturated material) and induced by gravity. (b) The movement of water in unsaturated soils due to capillary forces or water-film movements which are dependent upon capillary action of the soil, and the magnitude of these forces is controlled by physical and physico-chemical properties of soil.

Positive statement on the influence of the "capillary rise" on the position of phreatic line and upon the seepage flow through a homogenous earth dam may prove to be premature at this time. Selected references on these subjects are listed below.

References:

Physics of the Earth IX, Hydrology, edited by Oscar E. Meinzer

(a) Karl Terzaghi, Chapter IX Soil Moisture, "Soil Moisture and Capillary Phenomena in Soils," page 331

(b) Leonard D. Baver, "Retention and Movement of Soil Moisture," page 364

R. K. Schofield, "The pF of the Water Soil," Transaction of the Third International Congress of Soil Science, Oxford, England, 1935, page 37

Karl Terzaghi, "Theoretical Soil Mechanics," May 1944, Chapter XIV, page 297

Donald M. Burmister and Ek-Khoo Tan, Proceedings A.S.C.E., February 1948, Discussion, Stability of Soil Slopes, page 221

T. E. Buchrer and M. S. Rose, "University of Arizona College of Agriculture," June 20, 1943, Studies in Soil Structures versus Bound Water in Normal and Puddled Soils

N. J. Denisov, "Settlement Properties of the Loessial Soils." Government Publishing Office. "Soviet Science" U.S.S.R. Moscow, 1946 (Abstracted and translated by K. P. Karpoff and H. J. Gibbs, Bureau of Reclamation, Denver, Colorado, June 1951, Chapter IV, pages 7 to 9).

STABILITY OF SLOPES

The analysis of soil slope stability developed by N. N. Pavlovsky[4] is based on consideration of forces exerted by the percolating water. During the percolation through an earth dam, the water meets hydraulic resistance, and the flow will spend some part of its energy in overcoming the resistance. Such a condition exists in the dam, where the line of saturation intersects the downstream slope (M_o), and kinetic energy, not used to overcome resistance to flow, will exert a hydraulic pressure on the soil particles below M_o.

Pavlovsky's theory is that the hydraulic pressures which occur in the downstream region below the line of saturation of the earth structure have a definite effect on the stability of slopes and should be taken into consideration.

The initial failure of the downstream slope of an earth dam or earth canal slope as a result of ground-water pressure is usually localized in a definite region, i.e., below the intersection of the line of saturation with the slope, and this is followed by sliding of a mass of earth material above that region.

In this article is submitted a practical method used with good results in the field to compute the stability of slopes from a simplification by the author of N. N. Pavlovsky's theory.[5] The mathematical formulas involved and two illustrative numerical examples are given.

Discussion

After determining the position of the line of saturation in an earth dam by the method of N. N. Pavlovsky, discussed in the first part of this article, the stability of the downstream slope is studied by an analysis of the forces which act on a particle of soil at the outlet point of the saturation line.

Two forces act on a soil particle at the outlet point (Figure 5).

a. The weight of the particle which is directed vertically downward

b. The outward force of the seepage flow which has a direction tangential to the line of saturation at the outlet point

4. Pavlovsky, Professor N. N., "Filtration of Water through Earth Dam," Nauchno-Meliorasiony Institute, 1930, (in Russian) or thesis of K. P. Karpoff for B. S. in Civil Engineering, April 10, 1934 Engin. Library University of California.
5. Author's note: For simplicity of mathematical analysis a vector $\gamma_w J$ (Figures 5 and 6) was plotted along the slope; whereas Pavlovsky plotted a vector tangent ($\gamma_w J$) to the phreatic line at outlet point. By this simplification the final equation obtained was identical to Pavlovsky's equation.

These forces can be considered as acting on a unit volume of the soil material. The weight of this unit volume, in submerged unit weight, will be denoted by γ'; then, according to Terzaghi's formula,[6]

$$\gamma' = (s - 1)(1 - n) \qquad (16a)$$

where

s = specific gravity of soil material and
n = porosity of material

The outward force of the seepage water which acts on the unit volume will be

$$\gamma_w J$$

where J is the average hydraulic gradient for the unit volume, and γ_w is the weight of the unit volume of water.

If the resultant force R (Figure 6) of the two above-mentioned forces is projected on the plane of the slope and the plane normal to that slope respectively, the two components R_t and R_n will be obtained. Then the equation of equilibrium, at which the particle of soil will overcome the force of friction and will be ready to slip down the slope, may be written:

$$R_t = R_n \tan \phi \qquad (17)$$

where ϕ is the angle of internal friction of the soil material.

By projecting the force γ' on the plane normal to the slope (Figure 6), we obtain the vector R_n, and

$$R_n = \gamma' \cos \alpha \qquad (18)$$

Force $\gamma_w J$ and the component of γ' (Figure 6) on the plane of the slope, in a state of equilibrium, must be equal to the force of friction, Equation 17, and may be expressed:

$$R_n \tan \phi = \gamma_w J + \gamma' \sin \alpha \qquad (19)$$

After substituting in Equation 19 the value R_n from Equation 18 and solving for the limit of J, we have

$$J_{lim} = - \frac{\gamma'}{\gamma_w} (\tan \alpha - \tan \phi) \cos \alpha \qquad (20)$$

where

$$\frac{\gamma'}{\gamma_w} = \gamma''$$

then

$$J_{lim} = \gamma'' (\tan \phi - \tan \alpha) \cos \alpha \qquad (21)$$

6. Theory of Terzaghi--"Erdbaumechanik auf Bodenphysikalisher Grundlage," 1925, pp. 128-131.

According to Pavlovsky's theory, the value of the hydraulic gradient (J) at the outlet point M_o (Figure 7) of the line of saturation is equal to the slope of the dam; that is, it coincides with the slope of a structure, or in this case $\tan\alpha = \frac{y_o}{x_o}$ must represent a true gradient (J_{true}). In order for a soil material which lies on the slope to be stable, J_{lim} must be greater than J_{true}, or

$$\gamma''(\tan\phi - \tan\alpha)\ \cos\alpha > \tan\alpha \qquad (22)$$

Now introducing the safety factor "c" to Equation 22 we obtain

$$\gamma''(\tan\phi - \tan\alpha)\ \cos\alpha = c\tan\alpha \qquad (23)$$

This expression represents the largest possible value of hydraulic gradient for stability at the point under consideration.

The stability of an earth slope can be improved by placing a blanket of selected granular material on top of the slope, and in this case, Equation 23 is expressed as follows:

$$(\gamma'' + as')\ (\tan\phi - \tan\alpha)\ \cos\alpha = c\tan\alpha \qquad (24)$$

that is, the stability of slope is increased by $\gamma'' + as'$ or $1 + \frac{as'}{\gamma''}$ times where

a = the thickness of the layer of granular material (normal to the slope)

s' = bulk specific gravity of the granular material

Instead of computing $\tan\alpha$ by using Equation 24, it is possible to use a graph (Figure 8) which is plotted on the basis of Equation 24 where c and γ'' are considered equal to unity and the thickness of protective blanket from granular material a = o. Obtaining the value D^6 from the graph (Figure 8), we can determine the true value for the downstream slope of the structure by using Equation 25

$$\text{Slope} = \left[\frac{D}{\frac{c}{(\gamma'' + as')}}\right] : 1 \qquad (25)$$

For the practical application of the formula which is developed above, it is only necessary to have the following information about the soil:

Angle of internal friction (ϕ) of soil in a saturated condition
Specific gravity of soil
Average dry density
Bulk specific gravity of the protective granular material

Numerical Example

Given:
(1) Material, clay type
(2) Specific gravity - 2.71
(3) Angle of internal friction in a saturated condition $\tan\phi = 0.424$, $\phi = 23^o$
(4) Average dry density - 106.0 pounds per cubic foot
(5) Solid density = 62.4 x 2.71 = 169.1 pounds per cubic foot

7. $\frac{1}{D} = (\tan\phi - \tan\alpha)\ \cos\alpha$

Then porosity, $\underline{n} = \frac{(169.1 - 106.0)\ 100}{169.1} = 37.3$ percent, and weight per unit volume of soil at the hydraulic pressure

$$\gamma' = (s - 1)(1 - n) = (2.71 - 1)(1 - 0.377) = 1.07 \text{ grams per cubic centimeter and}$$

$$\gamma'' = 1.07$$

From the graph (Figure 8), for $\phi = 23^{o}$, the D = 4.75
Using 1.1 for the safety factor and introducing it in Equation 25, we obtain

$$\text{Slope} = \left[\frac{4.75 \times 1.1}{1.07}\right] : 1 = 4.88 : 1$$

Hence, the slope of the downstream face of the structure for the sake of stability should be about 5:1.

Let us see what will happen if a layer of selected granular material 0.458 meter is placed on this slope. Bulk specific gravity of the granular material (s') = 2.60, then substituting in Equation 25

$$\text{Slope} = \left[4.75\ \frac{1.1}{1.07 + 0.458 \times 2.60}\right] : 1 + 2.3 : 1$$

and the allowable slope of the structure with a protective blanket is about 2-1/4:1. This example shows clearly the utility of loading the lower portion of the downstream part of the earth dam with a pervious material.

Note: The safety factor c can vary from 1.1 and 1.3. The selection of a factor during the computation of slope depends upon the physical properties of soil material, field conditions, and the judgment of the designer.

It is believed that the above method, due to its simplicity and ease in mathematical approach, is of value for designers of earth hydraulic structures. It is particularly valuable to the field engineer where a quick decision must be made for determining the stability of a structure.

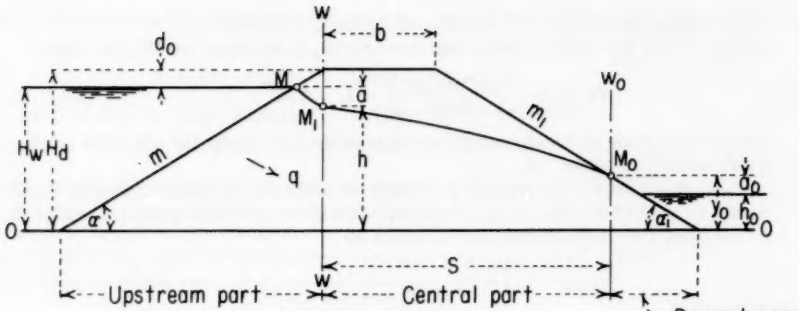


FIGURE 1

General conditions and nomenclature

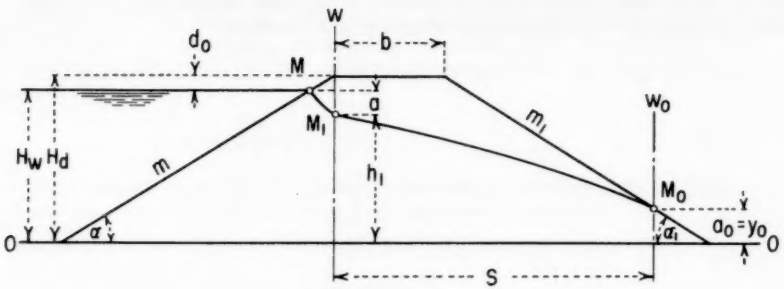
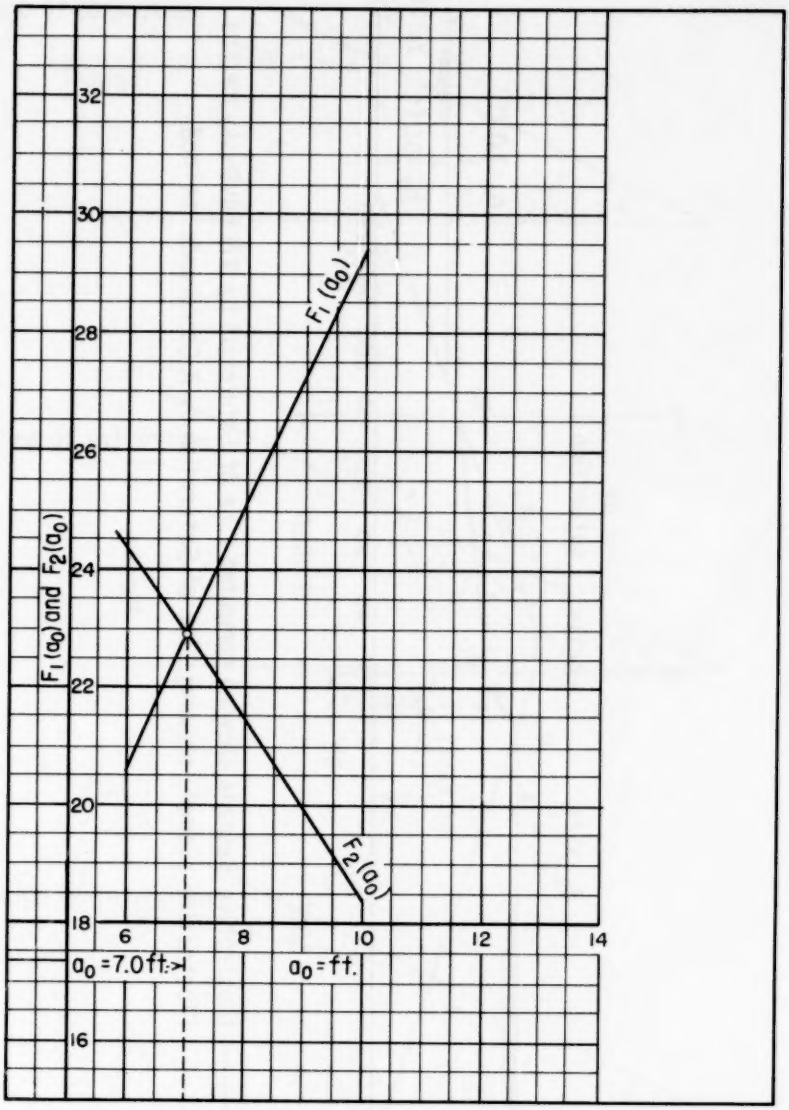


FIGURE 2

Special case when tail water is absent


32
30
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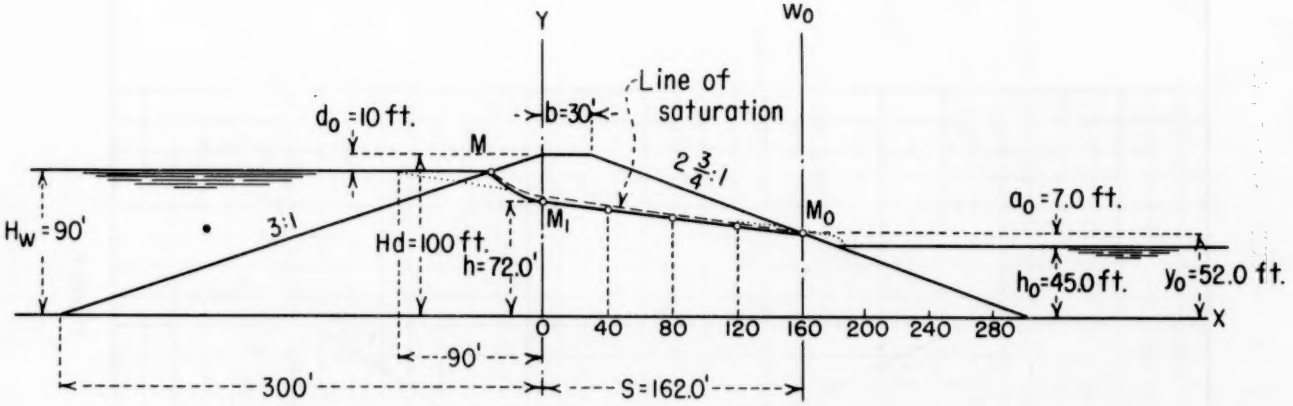
8
10
12
14
$F_1(a_0)$ and $F_2(a_0)$
$F_1(a_0)$
$F_2(a_0)$
a_0 = 7.0 ft.
a_0 = ft.

Figure 3



Illustrative example showing computed Line of Saturation by N.N. Pavlovsky method.

Illustrative example showing Line of Saturation by A. Casagrande method.

Figure 4

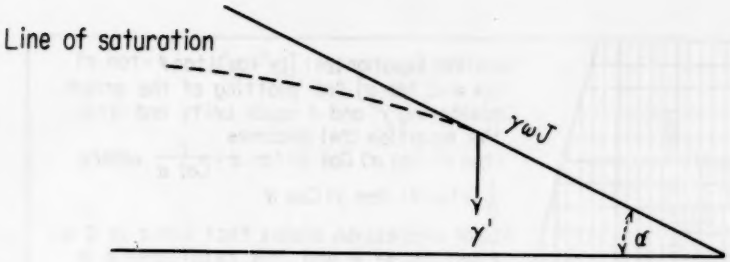


FIGURE 5

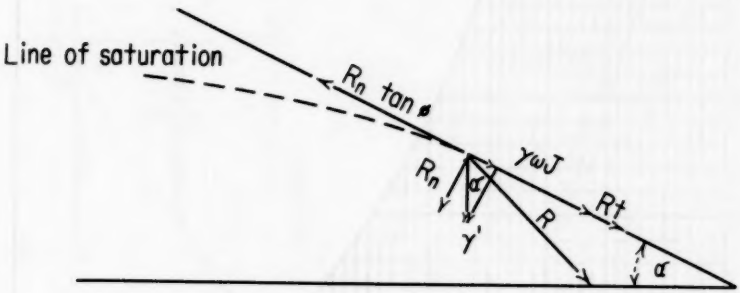


FIGURE 6

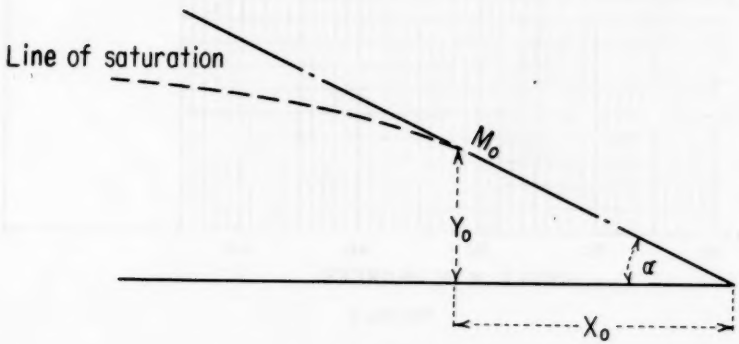


FIGURE 7

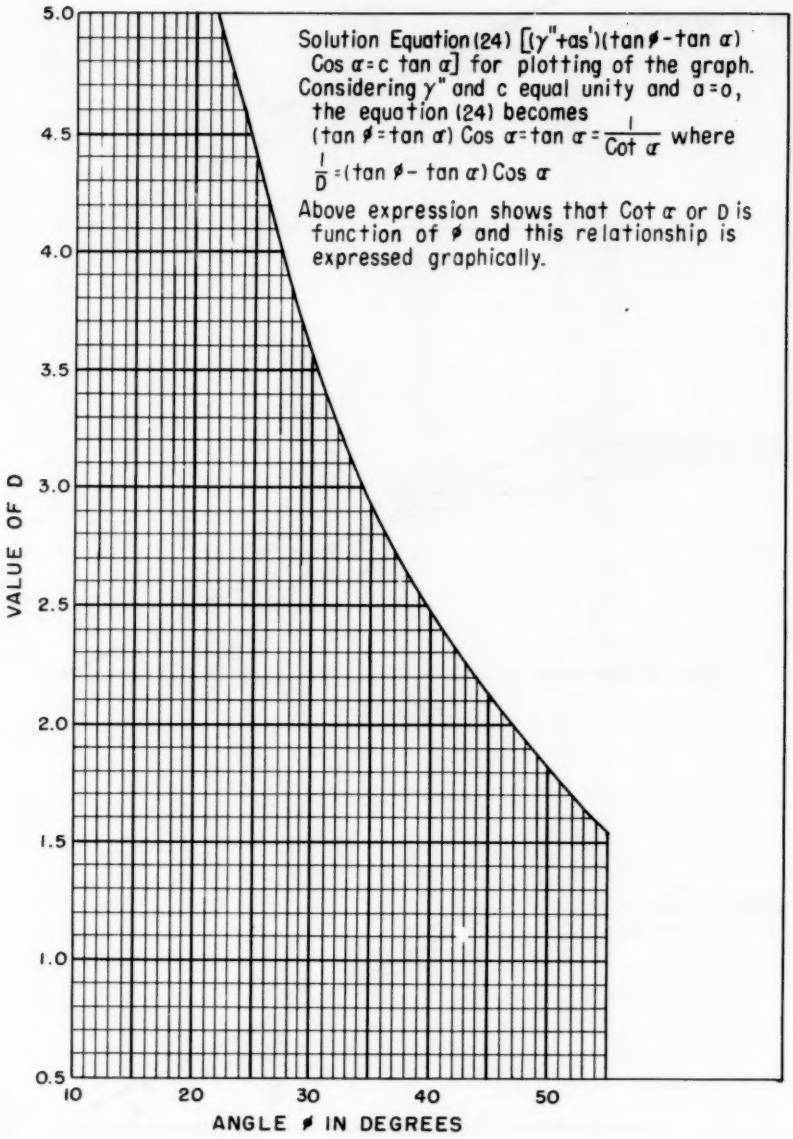
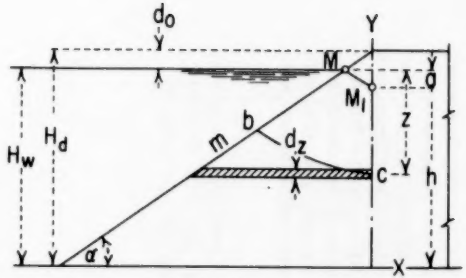
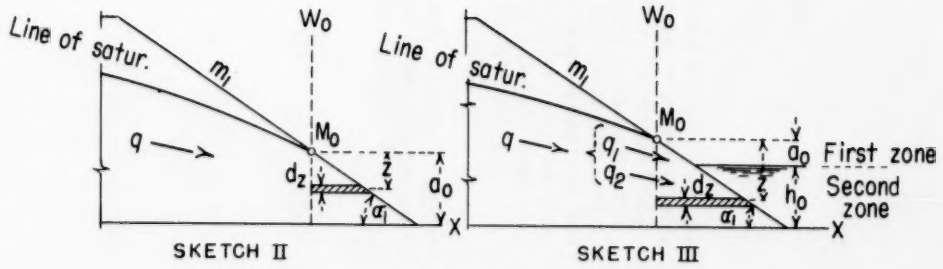

Solution Equation (24) [(γ"+as')(tan ϕ - tan α) Cos α = c tan α] for plotting of the graph.
Considering γ" and c equal unity and a = o, the equation (24) becomes
(tan ϕ = tan α) Cos α = tan α = 1/Cot α where
1/D = (tan ϕ - tan α) Cos α
Above expression shows that Cot α or D is function of ϕ and this relationship is expressed graphically.
VALUE OF D
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
10
20
30
40
50
ANGLE ϕ IN DEGREES

Figure 8



SKETCH I



SKETCH II SKETCH III

Figure 9

PROCEEDINGS-SEPARATES

VOLUME 80 (1954)

The technical papers published in the current calendar year are presented below. Technical division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering."

JANUARY: 379(SM),[a] 380(HY), 381(HY), 382(HY), 383(HY), 384(HY),[a] 385(SM), 386(SM), 387(EM), 388(SA), 389(SU),[a] 390(HY), 391(IR),[a] 392(SA), 393(SU), 394(AT), 395(SA),[a] 396(EM),[a] 397(ST).[a]

[a]Discussions grouped by Divisions.

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